VI. Conclusions

The theory presented in this paper results in a new method on how to design the transmission line of a matching network for a hybrid coupler phase shifter. The method uses the diode parasitic elements instead of tuning them out as suggested by Stark [3]. Formulas for optimization of phase shift versus frequency are presented. The theory gives the lower phase-shift error.

This paper also presents the formulas for the balance loss in the forward and reverse bias states. The balance loss gives the diminished loss variation for the phase shifter.

Acknowledgment

The author wishes to thank his dissertation adviser Prof. E. F. Bolinder for the support and encouragement he gave the author during this work. The author is also indebted to Dr. R. Ekinge, who introduced him to the field of phase shifters.

References

[1] J. F. White, "Diode phase shifters for array antennas," *IEEE Trans. Microwave Theory Tech.*, vol. 22, pp. 658–674, June 1974.
[2] R. W. Burns and L. Stark, "PIN-diodes advance high power phase shifting," *Microwaves*, pp. 38–48, Nov. 1965.
[3] L. Stark, "Microwave theory of phased-array antennas—A review," *Proc. IEEE*, vol. 62, pp. 1661–1701, Dec. 1974.

A Method for Broad-Band Matching of Microstrip Differential Phase Shifters

BURKHARD SCHIEK AND JÜRGEN KÖHLER

Abstract—**Meander-line phase shifters in microstrip are not well matched, because of the phase-velocity difference between the odd and the even mode in the coupled-line region. A stepped impedance design is introduced which allows one to realize, for example, a quarter-wavelength 90° Schiffman phase shifter with three sections, having, theoretically, a return loss better than 30 dB over a 1 : 5 bandwidth. Design equations are given and are confirmed by measurements on microwave integrated circuits in the frequency range 2–10 GHz.**

I. Introduction

ONE OF THE most useful passive components of microwave integrated circuits is the differential phase shifter, which is used most frequently for a phase shift of 45, 90, or 180°. Such differential phase shifters are conveniently constructed from parallel-coupled transmission lines connected at one end, as first described by Schiffman [1]. General synthesis procedures were first given by Steenaart [2] and Cristal [3].

The simplest circuit is the type A network [1] or microwave C section, shown in Fig. 1. The meander circuit of Fig. 1 will be perfectly matched at the input, independent of frequency, if the odd and even mode impedances Z_o and Z_e, respectively, conform to

$$Z_o \cdot Z_e = Z_0^2 \quad (1)$$

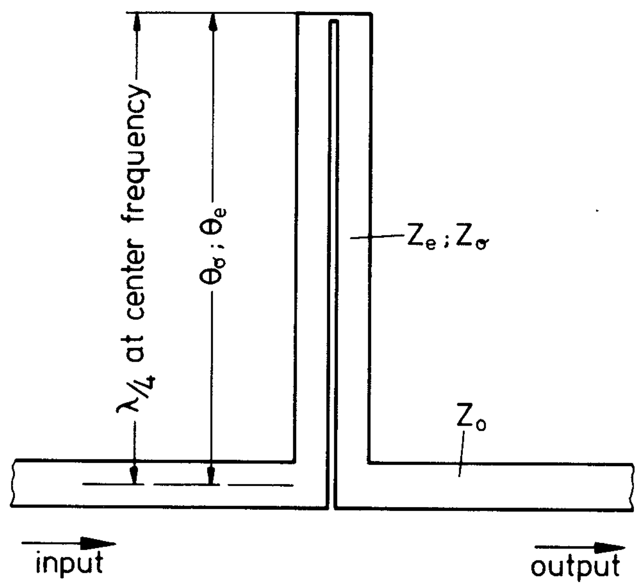

Fig. 1. Microwave C section or meander-line phase shifter visualized in microstrip.

(Z_0 is the characteristic impedance of the connecting transmission line) and if the electrical lengths of the odd and even modes, θ_o and θ_e, respectively, are equal:

$$\theta_o = \theta_e = \theta_i \quad (2)$$

i.e., ideal TEM behavior.

An example, to which we shall refer repeatedly in this paper, is an octave-wide 90° phase shifter which can be

Manuscript received October 11, 1976; revised January 20, 1977.
The authors are with Philips Forschungslaboratorium Hamburg GmbH, 2000 Hamburg 54 BRD, West Germany.

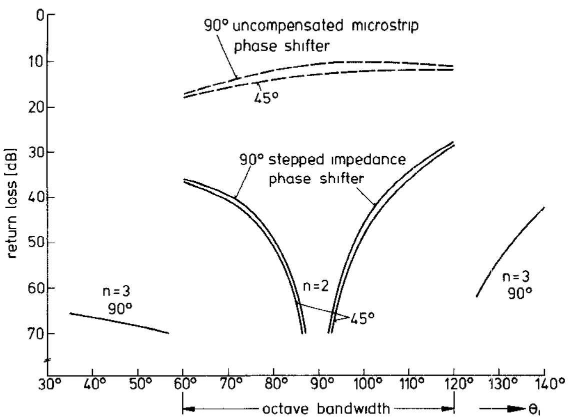

Fig. 2. Theoretical return loss of a 45° ($\rho = 1.7$) and a 90° ($\rho = 2.73$) microstrip phase shifter. The phase velocity of the odd mode is 12.3 percent higher than that of the even mode.

designed according to Fig. 1 with the ratio of $Z_e/Z_o = \rho = 2.73$ and a theoretical maximum phase error of $\pm 2.8°$. The reference line needs to have an electrical length of $3\theta_i$, i.e., $\frac{3}{4}\lambda$ at the center frequency [1].

Similarly, an octave-wide 45° phase shifter with a maximum phase error of $\pm 1.1°$ can be designed from the same structure as in Fig. 1 but with the ratio of $Z_e/Z_o = 1.7$ and a $\frac{5}{8}$ λ-long reference line.

These differential phase shifters, of either the single-stage type as in Fig. 1 or the multistage type [1]–[3], have been designed successfully on the basis of pure TEM transmission lines.

Trying to transpose these experiences to a microstrip circuit, it turns out that the meander circuit is badly matched due to a phase-velocity mismatch, i.e., $\theta_e \neq \theta_o$. Let us take the example of the 90° phase shifter with $\rho = 2.73$. Then, for a 50-Ω configuration on a 0.51-mm-thick alumina substrate the phase velocity of the odd mode is approximately 12.3 percent higher [4] than that of the even mode. This relatively small phase-velocity difference, however, strongly deteriorates the input matching of the meander as shown in Fig. 2. As can be seen from this figure, the theoretical return loss is around 10 dB, an intolerable value. The deterioration of the input matching is stronger for a meander line than for a parallel-coupled transmission-line directional coupler.

Although the phase velocities are unequal for coupled microstrip lines, they nevertheless show but little phase dispersion. Therefore we shall assume that θ_o and θ_e are directly proportional to the frequency. An additional assumption made in the following theory is that the phase velocities do not depend on the dimensions of the layout, i.e., on the magnitude of the odd- and even-mode characteristic impedances. This point shall be discussed in more detail in Section IV.

Podell has suggested a method of wiggling [5] in order to equalize the phase velocities. However, experiments which we have performed seem to prove that wiggling alone reduces the phase-velocity difference only slightly while introducing losses and phase dispersion.

In contrast we would like to propose a simple and convenient method suitable to match meander-line phase shifters across a broad frequency range [9].

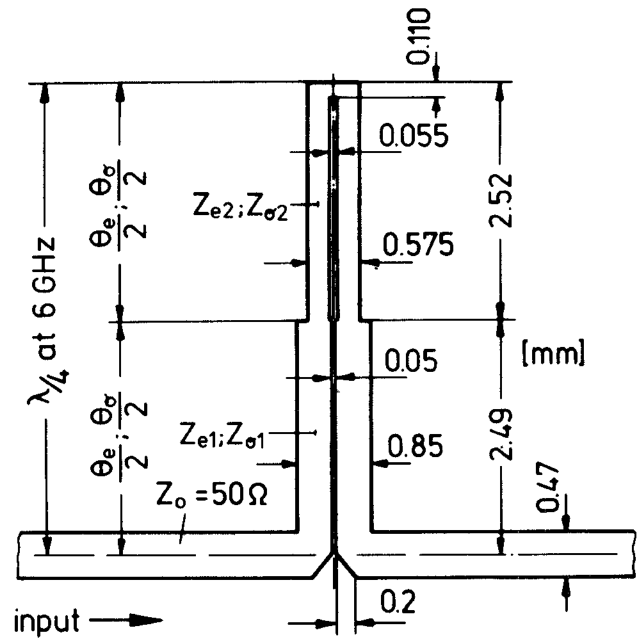

Fig. 3. A 90° two-section ($n = 2$) stepped-impedance meander-line phase shifter in microstrip technique. Layout for an alumina substrate 0.51 mm thick.

II. Formulation of the Problem

The meander-line phase shifter of Fig. 1 is most easily analyzed in terms of the input impedance Z_{in} for an even and odd excitation at the input and output port of the meander circuit. An even excitation produces an open circuit at the bend of the meander; an odd excitation, a short circuit. The respective input impedances are

$$Z_{\text{in}\,e} = -jZ_e \cot\theta_e, \qquad \text{even}$$

$$Z_{\text{in}\,o} = jZ_o \tan\theta_o, \qquad \text{odd.} \tag{3}$$

For equal phase velocities of the odd and even modes, i.e., $\theta_o = \theta_e$, the product $Z_{\text{in}\,e} \cdot Z_{\text{in}\,o}$ is real and independent of the frequency. This is the basis for a frequency-independent matching of the meander-line phase shifter. In the case of the microstrip phase shifter, the phase velocities of the odd and even modes differ considerably, and, therefore, the product $Z_{\text{in}\,e} \cdot Z_{\text{in}\,o}$ is no longer constant and independent of frequency. In particular, the zero of $Z_{\text{in}\,e}$ at $\theta_e = \pi/2$ does not coincide with the pole of $Z_{\text{in}\,o}$ at $\theta_o = \pi/2$. We will, therefore, try to shift the zero of $Z_{\text{in}\,e}$ to somewhat higher frequencies and the pole of $Z_{\text{in}\,o}$ to somewhat lower frequencies such that the zero and the pole coincide. This will be accomplished with a stepped-impedance structure as shown in Fig. 3. The coupled-line section of the meander is divided into two or more sections of equal electrical length (unit line sections) but with different characteristic-impedance values.

Let us first consider an approach with two sections ($n = 2$), each being approximately 45° long at the center frequency (Fig. 3). The frequency behavior of the structure of Fig. 3 should approximate the ideal TEM meander as closely as possible. The ideal meander, which serves as a prototype, has the same electrical length θ_i for the odd and even modes and the following input impedances:

$$Z_{ei} = -jZ_e \cdot \cot\theta_i, \qquad \text{even}$$

$$Z_{oi} = jZ_o \cdot \tan\theta_i, \qquad \text{odd.} \tag{4}$$

Somewhat arbitrary, the electrical length θ_i of the prototype is taken as the arithmetic mean of the odd and even modes, having electrical lengths θ_o and θ_e,

$$\theta_i = \frac{\theta_e + \theta_o}{2}. \tag{5}$$

At the center frequency, where $\theta_e = \theta_{ec}$ and $\theta_o = \theta_{oc}$ and $\theta_i = 90°$, we set

$$\frac{\theta_{ec} + \theta_{oc}}{2} = 90°. \tag{6}$$

The ratio

$$\frac{\theta_e}{\theta_o} = \frac{\theta_{ec}}{\theta_{oc}} \tag{7}$$

may be taken from tables [4] once Z_o and Z_e are known, and (6) and (7) allow θ_{oc} and θ_{ec} to be determined. For our example of a 90° phase shifter with $\rho = 2.73$ we found

$$\theta_{ec} = 95.2° \qquad \theta_{oc} = 84.8°. \tag{8}$$

III. General Synthesis Procedure and Examples

We set

$$t_{o,e} = j \tan \frac{\theta_{o,e}}{n} \tag{9}$$

$$\theta_e = \frac{\theta_{ec}}{90°} \cdot \theta_i \qquad \theta_o = \frac{\theta_{oc}}{90°} \cdot \theta_i. \tag{10}$$

Because in the following t_o is not used, t shall stand for t_e. Then the input impedance function of the two-section meander ($n = 2$) normalized to Z_e reads as follows for an even excitation:

$$z_{\text{in}\,e} = \frac{Z_{\text{in}\,e}}{Z_e} = \frac{t^2 + \alpha_{e1}}{\beta_{e1} \cdot t} = \frac{N}{D}. \tag{11}$$

The unknown coefficients α_{e1} and β_{e1} must be determined such that $z_{\text{in}\,e}$ approximates the ideal function $z_{ei} = -j \cdot \cot \theta_i$ (equation (4)) as closely as possible. One way to do this is to equate the first two terms of the Taylor series expansion:

$$z_{\text{in}\,e} = z_{ei} = 0, \qquad \text{at } \theta_i = 90° \tag{12}$$

$$\frac{d}{d\theta_i} z_{\text{in}\,e} \equiv \frac{d}{d\theta_i}\left(\frac{N}{D}\right) = \frac{d}{d\theta_i}(z_{ei}), \qquad \text{at } \theta_i = 90°. \tag{13}$$

The latter equation is more conveniently evaluated in the equivalent form

$$\frac{d}{d\theta_i}(Z_{ei} \cdot D) = \frac{dN}{d\theta_i}, \qquad \text{at } \theta_i = 90°. \tag{14}$$

The evaluation of (12) and (14) yields the coefficients α_{e1} and β_{e1}:

$$\alpha_{e1} = \tan^2 \frac{\theta_{ec}}{2} \qquad \beta_{e1} = \frac{\theta_{ec}}{90°} \cdot \cos^{-2} \frac{\theta_{ec}}{2}. \tag{15}$$

TABLE I

$z_{e1} = 0.94538$	$z_{o1} = 0.94222$
$z_{e2} = 1.13382$	$z_{o2} = 1.13004$

The normalized impedance values z_{e1} and z_{e2} (Fig. 3) are derived from the reactance function of (11) in a straightforward manner [6]:

$$z_{e1} = z_{\text{in}\,e}\big|_{t=1} = z_{e1}(t)_{t=1} = \frac{1 + \alpha_{e1}}{\beta_{e1}}. \tag{16}$$

Next, the first unit element with the characteristic impedance z_{e1} is eliminated from $z_{\text{in}\,e}$ and one obtains the input impedance $z_{e2}(t)$ of the remaining circuit

$$z_{e2}(t) = z_{e1} \frac{z_{e1}(t) - t \cdot z_{e1}}{z_{e1} - t z_{e1}(t)} = \frac{\alpha_{e1} \cdot z_{e1}}{t}. \tag{17}$$

The result on the right-hand side of (17) is obtained after a factor $1 - t^2$ has been cancelled in the numerator and denominator.

From (17) one obtains z_{e2} in the same manner as before

$$z_{e2} = z_{e2}(t)\big|_{t=1} = \alpha_{e1} \cdot z_{e1}. \tag{18}$$

For the odd-mode excitation exactly the same formulas apply for the characteristic admittances y. Therefore,

$$\alpha_{o1} = \tan^2 \frac{\theta_{oc}}{2}$$

and

$$\beta_{o1} = \frac{\theta_{oc}}{90°} \cdot \cos^{-2} \frac{\theta_{oc}}{2}, \qquad \text{odd mode} \tag{19}$$

and

$$y_{o1} = \frac{1 + \alpha_{o1}}{\beta_{o1}} \qquad y_{o2} = \alpha_{o1} y_{o1}. \tag{20}$$

If a two-section stepped-impedance meander-line phase shifter is designed according to the design equations (15), (16), and (18)–(20), it will be perfectly matched around the center frequency.

Two numerical examples will show, however, that the region of good matching may well cover an octave (Fig. 2).

The first numerical example concerns the 90° phase shifter mentioned before with $\rho = 2.73$ and a 50-Ω system. From tables of Bryant–Weiss [4] we evaluated θ_{ec} to be approximately 95.2° and θ_{oc} to be approximately 84.8°. The normalized characteristic impedances have been calculated from (15), (16), and (18)–(20) and are given in Table I. The characteristic impedances are obtained from Table I and the relations

$$Z_{e1,2} = z_{e1,2} \cdot \sqrt{\rho} \cdot 50\ \Omega$$

$$Z_{o1,2} = z_{o1,2} \cdot \frac{1}{\sqrt{\rho}} \cdot 50\ \Omega. \tag{21}$$

The return loss has been calculated and is also plotted in Fig. 2.

The theoretical return loss is above 28 dB across an octave bandwidth.

A final layout in microstrip on an alumina substrate 0.51 mm thick and for a center frequency of 6 GHz is shown in Fig. 3.

The second numerical example concerns a 45° phase shifter which needs a reference line of $\frac{5}{8}\lambda$ at the center frequency. The theoretical return loss for $n = 2$ is also plotted in Fig. 2. The values used for this calculation were $\theta_{ec} = 95°$, $\theta_{oc} = 85°$, and $\rho = 1.7$.

So far we have only discussed the return loss of stepped-impedance phase shifters but not their phase response. As is to be expected, the phase dependency is nearly identical to the ideal C section. The deviation is zero at the center of the frequency band. But even the maximum numerical deviations from the ideal C section are only 0.22° for the 90° phase shifter and 0.2° for the 45° phase shifter ($n = 2$) within an octave bandwidth.

The stepped-impedance phase shifter can be matched across an even wider frequency band, if the number of steps n is further increased. For the 90° phase shifter and $n = 3$ the return loss, theoretically, is better than 30 dB across more than a 1 : 5 bandwidth (Fig. 2).

The normalized impedance function for a three-section meander line is given by the expression

$$z_{\text{in}\,e} = \frac{\alpha_{e1} t^2 + \alpha_{e2}}{t^3 + \beta_{e1} t}. \tag{22}$$

From (22) one obtains the normalized characteristic impedances for the different sections (Fig. 4), by applying (16) and (17), with the following results:

$$z_{e1} = \frac{\alpha_{e1} + \alpha_{e2}}{1 + \beta_{e1}}$$

$$z_{e2} = \frac{\alpha_{e1} + 2\alpha_{e2} + \alpha_{e2}\beta_{e1}}{\alpha_{e1}\beta_{e1} - \alpha_{e2}} \cdot z_{e1}$$

$$z_{e3} = \frac{\alpha_{e2}(1 + \beta_{e1})}{\alpha_{e1} + \alpha_{e2}} \cdot z_{e2}. \tag{23}$$

Again, the same formulas apply for the odd-mode admittances. The coefficients α and β may be determined by equating $z_{\text{in}\,e}$ to the ideal function z_{ei} at $\theta_i = 90°$, as well as with the first and second derivative. For convenience the resulting equations for $n = 3$, obtained by equating the respective Taylor series expansions with the prototype function $Z_e = -j \cot \theta_i$, are also given:

$$\alpha_{e,o1} = \frac{3 \cdot 90°}{\theta_{e,oc}} \cdot \frac{2 \cdot \sin^2 \frac{\theta_{e,oc}}{3}}{1 - 2 \cdot \sin^2 \frac{\theta_{e,oc}}{3}}$$

$$\beta_{e,o1} = \tan^2 \frac{\theta_{e,oc}}{3} \left[1 + \frac{4}{1 - 2 \sin^2 \frac{\theta_{e,oc}}{3}} \right]$$

$$\alpha_{e,o2} = \alpha_{e,o1} \cdot \tan^2 \frac{\theta_{e,oc}}{3}. \tag{24}$$

TABLE II

$z_{e1} = 1.01023$	$z_{o1} = 1.01427$
$z_{e2} = 0.95015$	$z_{o2} = 0.93780$
$z_{e3} = 1.26349$	$z_{o3} = 1.23485$

The normalized impedances obtained from (24) for $n = 3$, $\theta_{ec} = 95.2°$, and $\theta_{oc} = 84.4°$ are given in Table II. For the special case of $\theta_{ec} = \theta_{oc} = 90°$ the values for the z's converge to one as expected.

Nearly the same numerical values as in Table II have been obtained by equating $z_{\text{in}\,e}$ with z_{ei} at three different but otherwise arbitrary frequencies within the band of interest. Then the coefficients α_1, α_2, and β_1 are obtained as the solution of a set of three linear equations.

In general, solving a set of linear equations seems to be more convenient, if four or even more sections are used. This becomes necessary if the meander is longer than a quarter wavelength, e.g., half a wavelength. An example for such a circuit will be given below.

If $n \geq 3$, the approximated function fits the prototype function so excellently across a very broad frequency range that it is only of little importance, where the sample points for the system of linear equations are taken. As a corollary to this it turned out that the determinant of the linear system was very small.

IV. Limitations of the Method Presented

The theory presented is founded on the assumption that the odd- and even-mode phase velocities differ from one another but do not depend on the magnitude of the characteristic impedance. However, in reality the phase velocity does change somewhat with the dimensions of the coupled microstrip. This can be accounted for by changing the length of the different sections according to the actual wavelength. However, there is the practical limitation that geometrically the sections for the odd and even modes have to be congruent.

Fortunately it may be observed (see, e.g., Tables I and II) that the ratio of the impedance values of the odd and even modes is nearly constant for the different sections. Therefore, the phase velocities of the odd and even modes will also change in the same direction. A good compromise then is to take a mean value between the odd- and even-mode wavelength, as has been done in the design of Figs. 3 and 4. Taking these mean values, the maximum deviation of the wavelength underlying the theory and the wavelength actually realized is not more than 1 percent, for both the odd and the even mode.

V. Experimental Results

A. A 90° Phase Shifter

The stepped-impedance C section has been built with $n = 2$ and $n = 3$ and the dimensions shown in Figs. 3 and 4. The measured transmission and return loss are plotted in Fig. 5. The return loss, which includes the two transitions from a coaxial line to microstrip, is mostly better than 20 dB.

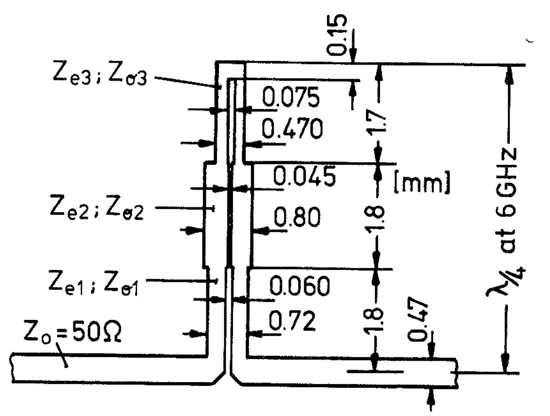

Fig. 4. A 90° three-section stepped-impedance phase shifter in microstrip technique. Layout for an alumina substrate 0.51 mm thick.

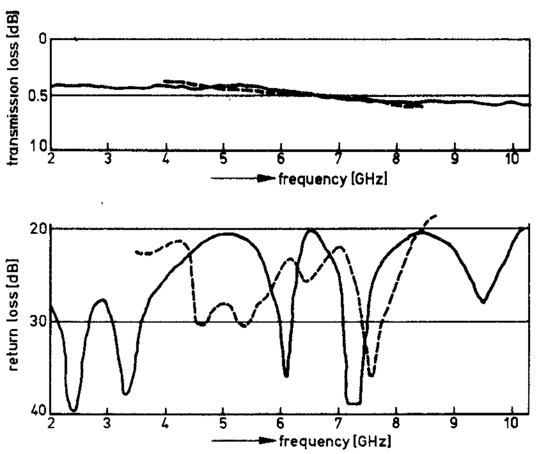

Fig. 5. Measured transmission and return loss of stepped-impedance phase shifters for $n = 2$ (dotted lines) and $n = 3$; layout as in Figs. 3 and 4.

End effects are very important. The even-mode excitation, corresponding to an open circuit at the bend of the meander, is capacitively loaded, while the odd mode is inductively loaded. These two effects should approximately compensate one another. Because a theoretical design procedure for the end effects is not yet available, a cut-and-try method has been used to design the bend of the meander (Figs. 3 and 4).

Another C section with $n = 2$ made for X band shows a return loss of better than 17 dB in the frequency range 7–12.4 GHz.

B. *A 180° Phase Shifter*

The phase difference between a half-wavelength uniform meander-line section and a reference line half a wavelength long is approximately 180°, if properly designed. The maximum phase errors within an octave bandwidth are $\pm 5.8°$ for a coupling of $\rho = 7.18$. This strong coupling value, however, cannot be realized in planar microstrip technique. Similarly, a quarter-wavelength-long meander line together with a wavelength-long reference line may form a 180° phase shifter. Also in this case the coupling is too tight to be realized in a planar form. Therefore the structure has been modified in so far as the reference line has also been folded to a meander line (Fig. 6). This increases the maximum phase error somewhat, but reduces the necessary coupling considerably. The prototype structure shown in Fig. 6 with the

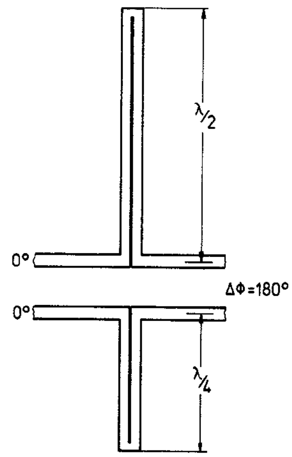

Fig. 6. A 180° phase shifter with the reference line also folded to a meander. $\rho = 2.62$ for both the half- and quarter-wavelength meander.

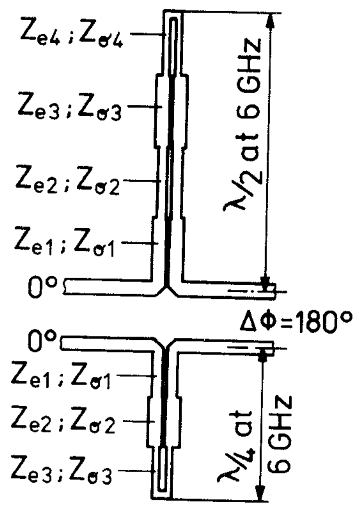

Fig. 7. Layout of an experimental 180° phase shifter. Normalized characteristic impedances are given in Tables II and III.

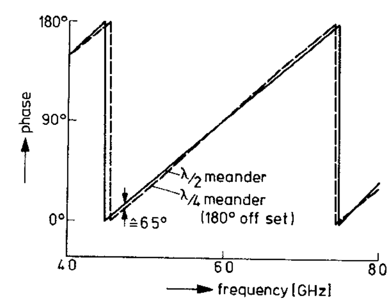

Fig. 8. Measured differential phase of the 180° phase shifter as given in Fig. 7 in the frequency range 4–8 GHz. The maximum measured phase error is $\pm 6.5°$. Arbitrary insertion length.

same coupling of $\rho = 2.62$ for both the half-wavelength and the quarter-wavelength meander is fairly close to optimum and has a maximum phase error over an octave bandwidth of $\pm 8.0°$. Fig. 7 shows the realization as a stepped-impedance structure and Fig. 8 the experimental results of the differential phase versus frequency obtained with this structure. The measured return loss was again better than 20 dB across the frequency range 4–8 GHz.

TABLE III

$z_{e1} = 0.9820$	$z_{o1} = 0.9757$
$z_{e2} = 1.0215$	$z_{o2} = 1.0370$
$z_{e3} = 0.8902$	$z_{o3} = 0.8703$
$z_{e4} = 1.3841$	$z_{o4} = 1.2847$

The half-wavelength stepped-impedance meander (Fig. 7) has been divided into $n = 4$ sections, sufficient to obtain matching across an octave bandwidth. The normalized characteristic impedances are given in Table III. They have been obtained numerically in a similar way as described above for $n = 3$. The unknown coefficients α_i and β_i were found as the solution of a linear set of equations.

The half-wavelength prototype meander may also consist of two cascaded quarter-wavelength sections with different characteristic impedances. This was named a type-F network [1]. In this case a stepped-impedance structure can be derived using exactly the same procedure as described above, except that the function to be approximated is different. The necessary number of sections n depends on the wanted bandwidth. We have not succeeded to apply our method for broad-band matching to higher order meander lines [7].

C. A Strip–Slot Realization of the C Section

Similar to a parallel transmission-line coupler which may be realized with a slot in the ground plane [8], a C section may also be realized with a strip on the top side and a slot in the ground plane of the substrate as shown in Fig. 9. In the figure, the dimensions given correspond to a coupling of $\rho = 5.83$. The strip–slot technique allows a stronger coupling to be realized in a planar design. A correction for the different phase velocities of the slot and strip lines can be accomplished simply by making the slot line somewhat longer than the strip line. This has been done in an experimental unit designed according to Fig. 9. The measured return loss was better than 18 dB in the frequency range 3.5–9 GHz. Of course, also the stepped-impedance concept can be used for the design of the strip–slot C section.

VI. Conclusion

Microwave constant differential phase shifters are useful passive components for the design of microwave circuits. These phase shifters are very conveniently constructed from folded transmission lines, i.e., meander lines. These meander lines are badly matched in the microwave integrated circuit technique with its non-TEM behavior, due to the difference in phase velocities of the odd and even modes. With a

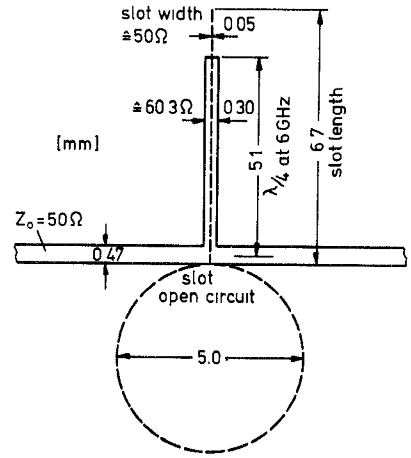

Fig. 9. Strip-line–slot-line realization of a microwave C section. Corresponds to $\rho = 5.83$ or 3-dB coupling. Layout for an alumina substrate 0.51 mm thick.

stepped-impedance procedure which has been introduced in this paper, a microstrip meander-line phase shifter may be designed from TEM all-pass networks as prototypes. For all practical purposes such a stepped-impedance microstrip meander is equivalent to its pure meander-line counterpart.

Acknowledgment

The authors wish to thank Dr. W. Schilz for fruitful discussions.

References

[1] B. M. Schiffman, "A new class of broadband microwave 90-degree phase shifters," *IRE Trans. Microwave Theory Tech.*, pp. 232–237, Apr. 1958.

[2] W. J. Steenaart, "The synthesis of coupled transmission line all-pass networks in cascade 1 : *n*," *IEEE Trans. Microwave Theory Tech.*, pp. 23–29, Jan. 1963.

[3] E. G. Cristal, "Analysis and exact synthesis of cascaded commensurate transmission-line C-section allpass networks," *IEEE Trans. Microwave Theory Tech.*, pp. 285–291, June 1966.

[4] T. G. Bryant and J. A. Weiss, "Parameters of microstrip transmission lines and of coupled pairs of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, pp. 1021–1027, Dec. 1968.

[5] A. Podell, "A high directivity coupler technique," in *G-MTT 1970, Int. Microwave Symp.*, pp. 33–36, May 1970.

[6] A. Matsumoto, "Microwave filters and circuits," in *Advances in Microwaves* (Supplement 1). New York: Academic Press, 1970, p. 20.

[7] S. Rehnmark, "Wideband planar transmission line PIN-diode attenuator/modulator," in *Proc. European Microwave Conf.* (Hamburg, Sept. 1975), pp. 128–132.

[8] B. Schiek, "Hybrid branchline couplers—A useful new class of directional couplers," *IEEE Trans. Microwave Theory Tech.*, pp. 865–869, Oct. 1974.

[9] B. Schiek, J. Köhler, and W. Schilz, "Broadband matching of microstrip differential phase shifters," in *Proc. European Microwave Conf.* (Rome, Sept. 1976), pp. 647–651.